

# Demosaicing as the Problem of Regularization

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## ABSTRACT

Demosaicing is the process of reconstruction of a full-color image from Bayer mosaic, which is used in digital cameras for image formation. This problem is usually considered as an interpolation problem. In this paper, we propose to consider the demosaicing problem as a problem of solving an underdetermined system of algebraic equations using regularization methods. We consider regularization with standard  $l_{1/2}$ -,  $l_1$ -,  $l_2$ -norms and their effect on quality image reconstruction. The experimental results showed that the proposed technique can both be used in existing methods and become the base for new ones.

**Keywords:** Digital camera, Bayer mosaic, demosaicing, interpolation, regularization.

## 1. INTRODUCTION

Today most digital cameras form color images using a monolayer matrix of photocells. Every element of this matrix is covered with some color filter of a fixed set that transmits light of a specific spectral composition. These filters form a mosaic in the matrix. The most common arrangement of color filters is the Bayer mosaic [1], which consists of 25% red, 25% blue and 50% green elements arranged as shown in Figure 1(a). Thus, each pixel contains information about only one of the three color components (Figure 1(b)) while the color image contains all three components for each pixel. The problem of estimating missing color components is called demosaicing or debayering.

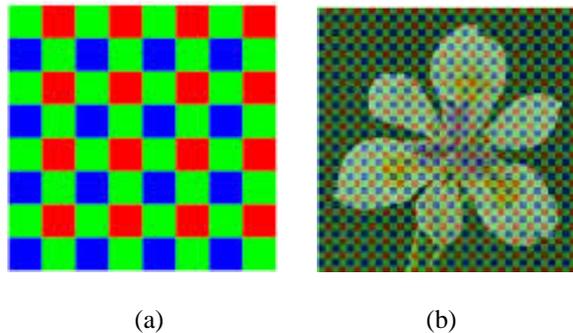


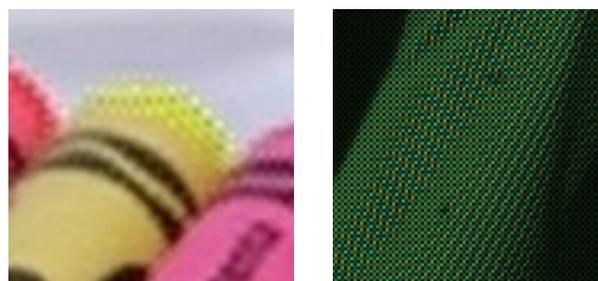
Figure 1: (a) The arrangement of the color filters in the Bayer mosaic (b) visualization of an image filtered by the Bayer mosaic

## 2. EXISTING METHODS

The existing debayer methods can be divided into 2 groups: restoring the missing values of color components for each channel separately and using the correlation between the color channels.

Methods for interpolating each channel individually are examples of multivariate interpolation on a uniform grid that use adjacent known values of the same color channel as interpolation nodes. The nearest neighbor interpolation is the simplest method, which simply copies the closest component of the same color channel to the interpolated one. The bilinear interpolation is another simple method, where the value of the missing components is calculated as the average of its neighbors. More sophisticated interpolation techniques include adaptive methods, which use different techniques of interpolation, depending on the area around the pixel [2–4].

Although these methods can obtain good results in homogenous image regions, they are prone to severe demosaicing artifacts in regions with a sharp color transition or highly textured objects [5], such as zipper-effect, showed in Figure 2(a), which appears on the boundaries of areas of different colors as alternate values pixels of different color shades, and color moire, showed in Figure 2(b), characterized by an undulating pattern which is not present on the object.



(a) (b)  
Figure 2: (a) Zipper-effect (b) Color moire

Inter-channel correlation methods are also susceptible to artifacts, but in a lesser degree. They are characterized by a smooth transition of color. These methods often restore the green channel (the most representative) first and then the red and the blue ones, based on certain assumptions about the correlation between the color channels and using the green channel as the base one [6–10]. One of the possible assumptions is based on the fact that the ratio between the primary color components (for example, the ratio of R / G and B / G) remains constant within the object borders in the image [6, 7]. This assumption follows from the linear model of the color image forming [11]. The quality of such methods can be improved, if the initial restoration quality of the green channel is improved. One can use interpolation techniques such as NEDI [8] or those, based on the theory of optimal recovery, proposed in [9], which is a NEDI extension that adds some restrictions on the values of interpolated pixels. One of these methods is [10], which uses a combination of [6–9] and picks out a method, depending on the complexity of the image texture.

### 3. DEBAYERING AS A REGULARIZATION PROBLEM

We propose to consider the debayering process not as a problem of interpolation, but as a problem of image reconstruction using regularization methods.

It can be formulated as a linear inverse problem:

$$Af = u, \tag{1}$$

where  $f$  is the reconstructed image of size  $N \times M$  pixels,  $u$  is the source image,  $A$  is a linear operator on matrix  $f$  so that  $A: f \rightarrow u$ ;  $f$  is to be estimated. This rectangular matrix of  $A$  is the concatenation of the unit and zero matrices. The problem of the form (1) is ill-posed problem, and its solution is fundamentally unstable. Strictly speaking, half of the vector  $f$  components is not involved in the formation of the observed image for the problem of estimating the green channel. Note that there are no fundamental differences from the more usual case when variables are interrelated, but the number of equations is not enough. Therefore, to overcome the instability, one can use a regularization approach [12, 13], which takes a prior information of the desired image in algebraic account. Among the methods of regularization a special mention should go to Tikhonov method [12], as it leads solution structure analysis to nothing more than the OLS method and therefore it has a wider application in solving ill-posed inverse problems, such as computer tomography problem with a small number of projections.

In the case of Tikhonov regularization, equation (1) is solved by minimizing the following functional:

$$R(f) = \|Af - u\|_2^2 + \alpha \|Lf\|_2^2, \tag{2}$$

where  $L$  is an arbitrary linear operator that is selected, based on a prior information, and  $\alpha$  is a regularization parameter.

Regularization that limits the amplitude of the signal is effective in many applications. In the simplest case  $L$  is considered to be identity matrix. But it is obvious that in this case it would adjust to zero all the unknown values. However, an image is a piecewise continuous function with rare discontinuities. Hence, it makes sense to limit the gradient of an image. In this case, the problem (2) can be rewritten as follows:

$$R(f) = \|Af - u\|_2^2 + \alpha(\|f_x\|_2^2 + \|f_y\|_2^2), \quad (3)$$

where  $f_x$  and  $f_y$  are two images, where each pixel contains approximate derivatives with respect to  $x$  and  $y$ .

This is where the choice of metrics appears. It is known that  $l_2$  minimization applied to the problem of image reconstruction leads to object boundaries blurring in the image. It would seem that one can use  $l_0$  norm, the minimum of which is equivalent to maximum zero values of unknown components from (3). However, in the case of solving (1), its use would lead to an NP-complete problem and its solution is possible only for small data. Therefore, as an alternative, it was decided to use the  $l_1$ -norm which is successfully used in compressed sensing [14] and which results are comparable to  $l_0$ . This technique is called Total Variation (TV) [15–17].

Now, let's consider using (3) in terms of debayering. Then the problem of estimating the unknown components can be formulated as an optimization problem of the following form:

$$f = \arg \min R_p(f) \text{ subject to } Au = f, \quad (4)$$

where  $R_p(f)$  is a regularizing function minimized with  $l_p$ -norm over the image  $f$ .

Let's define  $R_p(f)$  as follows:

$$R_p(f_c) = \sum_{k=1}^K |f_c - f_k|^p, \quad (5)$$

where  $f_c$  is the central component in the  $3 \times 3$  window, the value of which is to be estimated by means of  $f_k$  which are known.

It is clear that using  $l_2$  norm, the minimum of (5) can be reached by averaging of components  $f_k$ :

$$R'_2(f_c) = 2 \sum_{k=1}^K (f_c - f_k) = 0 \quad \text{and} \quad f_c = \frac{\sum_{k=1}^K f_k}{K},$$

which brings us to the bilinear interpolation method that is base demosaicing method.

If we consider (5) using  $l_1$ -norm it is easy to prove that the minimum is the range of  $[f_{\frac{K}{2}-1}, f_{\frac{K}{2}+1}]$  where  $f_k$  components are known and sorted in some order. The definition of  $f_c$  can be extended in a standard way such that  $f_c = \frac{1}{2}(f_{\frac{K}{2}-1} + f_{\frac{K}{2}+1})$  that similar to the effect median filter.

Recently, as an alternative to  $l_1$  and  $l_0$  norms,  $l_{1/2}$  regularization was proposed [18, 19]. However, the  $R_{1/2}(f_c)$  minimization leads us to the problem of non-convex optimization, so its value can only be measured experimentally in the problem of restoring the image. Using the values of  $f_k$  in the neighborhood of  $f_c$  and assuming the fact that pixel

values are integer, in our case, in order to find the global minimum, an exhaustive search of possible values of  $f_c$  in the range  $[f_l, f_K]$  can be applied.

#### 4. EXPERIMENTAL RESULTS

For our experiments we used standard Kodak and IMAX datasets. The experiments were carried out on the green channel, the restoration of which has traditionally been considered to be the base problem of demosaicing, using  $l_{1/2}$ -,  $l_1$ -,  $l_2$ -norms, which were mentioned above, and  $l_{p_v}$ -norm of the following type:

$$l_{p_v} = \frac{I}{I + (f_x + f_y)}, \tag{6}$$

where  $f_x, f_y$  are vertical and horizontal gradients which are calculated using neighbors of the interpolated pixel.

To measure the quality of the green channel recovery, the reconstructed images was compared with the corresponding source images using standard metric *PSNR* (peak signal-to-noise ratio). *PSNR* is defined using the mean squared error (*MSE*). For a  $N \times M$  monochrome source image  $I$  and its approximation  $\hat{I}$ , *MSE* is defined as:

$$MSE = \frac{1}{NM} \sum_n \sum_m |I_{nm} - \hat{I}_{nm}|^2. \tag{7}$$

The *PSNR* is defined as:

$$PSNR = 10 \log_{10} \left( \frac{255^2}{MSE} \right). \tag{8}$$

We compared the results of image reconstruction using various norm regularization with standard bilinear interpolation method, which is a special case of regularization methods as it was mentioned above. Choosing different types of regularization operators we can determine the most effective one. Table 1 illustrates the average quality of the restoration of the green channel with  $l_{1/2}$ -,  $l_1$ -,  $l_2$ - and  $l_{p_v}$ -regularization for Kodak and IMAX datasets. The comparison shows the advantage of  $l_1$ - and  $l_{p_v}$ -metrics over  $l_2$ -metric.

Table 1: Quality of green channel restoration using regularization of Kodak and IMAX datasets

Average quality of the image restoration for dataset	PSNR			
	$l_2$	$l_1$	$l_{1/2}$	$l_{p_v}$
Kodak	33.6748	33.8563	31.8879	33.8571
IMAX	34.6054	36.1131	33.0601	36.1131

#### 5. CONCLUSION

We applied a regularization approach to demosaicing. This proposed technique takes into account the smoothness of color components in the same color channel. In this work, we consider a regularization with  $l_{1/2}$ -,  $l_1$ -,  $l_2$ -norms and elastic  $l_{p_v}$ -norm, which was mentioned above. For our experiment, we used Kodak and IMAX datasets. Although the solution of this problem as a problem of non-convex optimization didn't show good result, in general the proposed

technique showed improvement in the PSNR as compared to the bilinear method which is similar to  $l_2$  - regularization. Therefore, the proposed simple technique can push itself as a candidate for many applications of demosaicing methods.

The next possible step is the development of a structure of the regularization function that would take into account the correlation between color channels and use an appropriate norm regularization according to a prior information about image nature.

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